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Abteilung A: Versicherungswirtschaft

**The effects of probabilistic insurance on the
capital budgeting decision in respect of a value
based management**

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Mitteilungen

1/2005

Abstract

As empirical studies show, the policyholders' willingness to pay depends on the security level of the insurance cover. The choice of the security level determines the capital budgeting decision and therefore the cost of capital, as well as the attainable insurance premiums. We call this cost of capital-effect and premium-effect. Hence, the choice of the security level seems to be an important aspect of value based management. In this context, the description of the policyholders' willingness to pay in dependence of the security level is of special interest. In particular *Wakker/Thaler/Tversky* (1997) develop an explanation of the policyholders willingness to pay for probabilistic insurance based on Prospect Theory. In our paper, we describe and compare the cost of capital-effect and the premium-effect for varying security levels, supported by an example. Under simplifying assumptions, our pragmatic model determines the value proposition of different security levels. Therefore, it generates important knowledge for value based management.

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1. Introduction

Beside a risk adequate premium, insurance companies need a risk adequate capital. So, capital affects the underwriting limits and is a quality measure for the insurance cover at the same time. For investors, capital is a benchmark for the assessment of the profitability of their investments. The investors demand future cash flows which overcompensate the cost of capital.

According to its practical meaning, the capital budgeting decision is intensely discussed in the scientific literature. Nevertheless, most of these papers focus the choice of risk measures.¹ Other aspects, in particular the security level, are commonly accepted as given. As empirical studies show, the policyholders' willingness to pay depends on the security level of the insurance cover. Hence, the choice of the security level seems to be an important aspect of value based management.

Existing papers that examine the link between capital budgeting and probabilistic insurance are typically based on a neoclassical framework. E.g. *Gründl/Schmeiser* (2002) determine the value maximizing capital structure of insurance companies regarding the effects of probabilistic insurance on the policyholders willingness to pay. *Cummins/Sommer* (1996) develop a profit maximization model based on option price theory, that integrates the demand of insurance as a function of the insolvency put option.²

To describe the policyholders' willingness to pay in dependence of the security level, scientific literature also uses decision theory. Because of their empirical validation descriptive decision models - in particular the Prospect Theory by *Kahneman/Tversky* (1979) - have advantages relative to normative models, such as the Expected Utility Theory by *Neumann/Morgenstern* (1947).³ Above all *Wakker/Thaler/Tversky* (1997)

¹ See for instance Albrecht 1998; Albrecht/Koryciorz 2000; Artzner 1999; Barth 2000; Butsic 1994; Koryciorz 2004; Nakada et al. 1999; Panning 1999.

² For a discussion see Schradin 2004, pp. 800.

³ See Albrecht/Maurer 2000, pp. 344; Kahneman/Tversky 1979, p. 270; Theil 2002, pp. 52; Wakker/Thaler/Tversky 1997, pp. 8.

develop an explanation of the policyholders willingness to pay for probabilistic insurance based on Prospect Theory.

To sum up, the choice of the security level determines the capital budgeting decision and therefore the cost of capital, as well as the attainable insurance premiums. We call this cost of capital-effect and premium-effect. So, an analysis of these effects for varying capital budgets seems to be relevant for a value based management. Nevertheless, we don't know a discussion of these aspects based on decision theory. So, the present work will describe and face these effects, supported by an example.

2. The security level as an object of decision making

Under compliance of supervisors' minimum request, the capital budget is an important variable for the value based management. Alternative capital budgets have to be linked with expected future cash-flows. For this purpose, we can concentrate on such cash-flows that are determined by the capital budget. These are in particular the cost of capital and the premiums.⁴ The profitability of capital budgets exceeding the supervisory minimum requirement is given if the additional premiums exceed the additional cost of capital.

The cost of capital reflects the minimum return the investors expect for their investment. If the insurance company is in possession of a holding or a well defined investors group, the company may be confronted with a clearly defined expectation of return. In case of many investors such an explicit hurdle rate is not available. Hence, the cost of capital need to be determined by adequate models, such as the Capital Asset Pricing Model (CAPM) by *Sharpe* (1964), *Lintner* (1965) and *Mossin* (1966) or the Arbitrage Pricing Theory (APT) by *Ross* (1976; 1977).⁵

As mentioned, the capital budget is a quality measure for the insurance cover. Only under condition of the expectation of higher premiums a rational insurance company will spend money for quality improvement. But the policyholder value depends on the

⁴ Besides the capital budget is available to investment purposes, which generate expected future cash-flows as well. For reasons of simplification we don't regard this effect.

⁵ See Liebwein 2005, p. 323.

security level rather than on capital budgets. However, the security level depends on other factors, such as reinsurance and underwriting policy. Therefore, the following analysis bases on a *ceteris paribus* assumption.

Empirical studies confirm the link between the policyholders' willingness to pay and the insurers' security level. In this context *Wakker/Thaler/Tversky* (1997) questioned 86 students concerning their willingness to pay for a fire insurance. Standard Insurance (SI) denotes the case, where the policyholder is fully reimbursed if a hazard occurs. Analogous, probabilistic insurance involves the possibility that the claim won't be reimbursed. Given the probability of 0.5% for an insured loss of \$125.000, the average willingness to pay was \$700. In case of a default probability of 1%, the average willingness to pay decreased to \$500. In a second step, *Wakker/Thaler/Tversky* (1997) raised the insured loss to \$250.000. The average willingness to pay increased to \$1.300 for standard insurance and to \$900 for a probabilistic insurance given a failure probability of 1%.⁶

To quantify the value proposition of changes in capital budget, the insurance management needs an accurate description of the links between the security level and the cost of capital on the one hand and the policyholders' willingness of pay on the other hand. Therefore, the insurance management has to identify the cost of capital-effect and the premium-effect as accurately as possible.

3. The cost of capital-effect

Several different models for determination of risk adequate capital compete. Models based on risk theory need a reliable and sophisticated analysis, a modelling and quantification of all risks and their dependencies over time. These are essential challenges for insurance management, in particular in insurance classes with low frequency but high severity claims.

⁶ See *Wakker/Thaler/Tversky* 1997, pp. 8. See also the studies of *Albrecht/Maurer* 2000, pp. 344; *Kahneman/Tversky* 1979, p. 270; *Theil* 2002, pp. 52.

A huge number of different risk measures is available for the determination of risk adequate capital. However, there is a special focus on Value-at-Risk (VaR) and recently on Tail Value-at-Risk (TVaR).⁷

The $(1-\varepsilon)$ quantile of the loss distribution, the Value-at-Risk (VaR), states the critical height of loss (L) which is exceeded with a probability of ε . Hence, it is the capital requirement to survive the next period with the probability of $(1-\varepsilon)$. For cumulative distribution functions F of the insured loss the Value-at-Risk is defined as:

$$(1) \quad \text{VaR}_\varepsilon = F^{-1}(1-\varepsilon).$$

The Tail Value-at-Risk determines the necessary capital as the conditional expectation of the loss heights beyond an accepted quantile. Hence, it corresponds to the expected loss of ε 100% worst cases. Also for continuous loss distributions it is defined as:

$$(2) \quad \begin{aligned} \text{TVaR}_\varepsilon(L) &= \text{VaR}_\varepsilon(L) + E[L - \text{VaR}_\varepsilon(L) | L > \text{VaR}_\varepsilon(L)] \\ &= E[L | L > \text{VaR}_\varepsilon(L)] \end{aligned}$$

The preceding equation shows that the Tail Value-at-Risk always exceeds the Value-at-Risk for the same confidence level $(1-\varepsilon)$.

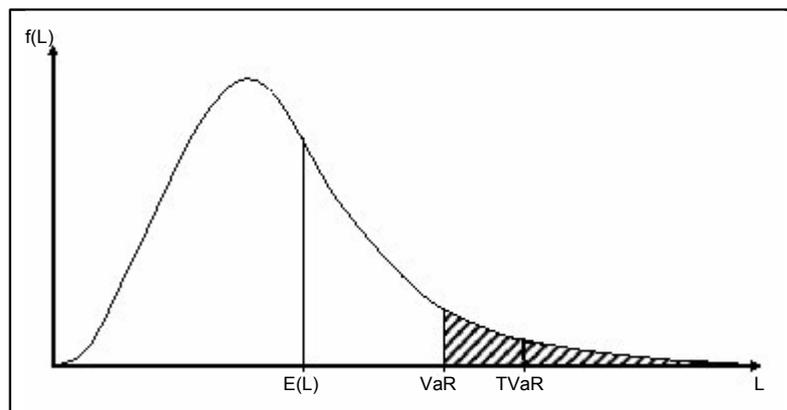


Figure 3-1: Value-at-Risk vs. Tail Value-at-Risk

⁷ See for instance European Commission 2003, pp. 33.; IAA 2004 pp.34; Koryciorz 2004 and Panjer 2002.

Because of insurance supervision, we focus small ruin probabilities (ε). Hence, little changes of the desired security level have great effects on the risk adequate capital. In the following, this is demonstrated for two loss distributions.

First, we have to identify the loss distribution. To keep it simple, the model just involves premiums (π) and claims (S). Hence, the losses are determined by:

$$(3) \quad L = S - \pi.$$

The premiums are given by 10 Mio. EUR while the claims are log-normal distributed with $S \sim \text{LN}(m, v^2)$. We are using the log-normal distribution because it's the distribution for amount of claims with the highest empirical evidence.⁸ So the Value-at-Risk is defined as:⁹

$$(4) \quad \text{VaR}_\varepsilon(L) = \exp(m + N_{1-\varepsilon} v) - \pi,$$

with $N_{1-\varepsilon}$ as the $(1-\varepsilon)$ quantile of the standard-normal distribution. The Tail Value-at-Risk is defined as:¹⁰

$$(5) \quad \text{TVaR}_\varepsilon(L) = E(S) \frac{1 - \Phi(N_{1-\varepsilon} - v)}{\varepsilon} - \pi,^{11}$$

with $\Phi(\cdot)$ as the cumulative distribution function of the standard-normal distribution.

Let's assume that the first log-normal distribution has the parameters $m_1=1.5$ and $v_1=1$, whereas the second is given by $m_2=0.5$ and $v_2=1.2^2$. Figure (3-2) shows the resulting density functions.

⁸ See Lippe 1983, S. 50; Mack 2002, S. 69. The disadvantage of the log-normal distribution is a bad approximation of extreme claim sums. See Lippe 1983, S. 50. Nevertheless, for determination of risk adequate capital, the right tails are of high interest. Anyway, we use the log normal distribution, because losses of insurance companies are mostly right-skewed distributed.

⁹ See Koryciorz 2004, p. 81.

¹⁰ See again Koryciorz 2004, p. 81. Koryciorz uses the term Conditional Value-at-Risk instead of Tail Value-at-Risk.

¹¹ The expected claim $E(S)$ is determined by $E(S) = \exp(m + \frac{v^2}{2})$.

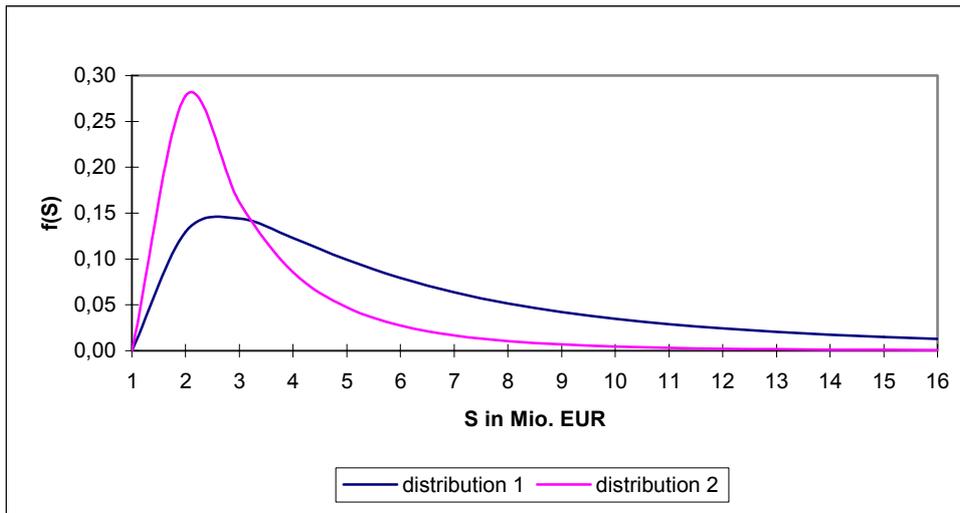


Figure 3-2: Density functions

The first distribution is more dangerous than the second, because the probabilities of high severity claims are considerably higher. That results in higher capital requirements for distribution one. The capital requirement as a function of ruin probability is shown in the following figure.

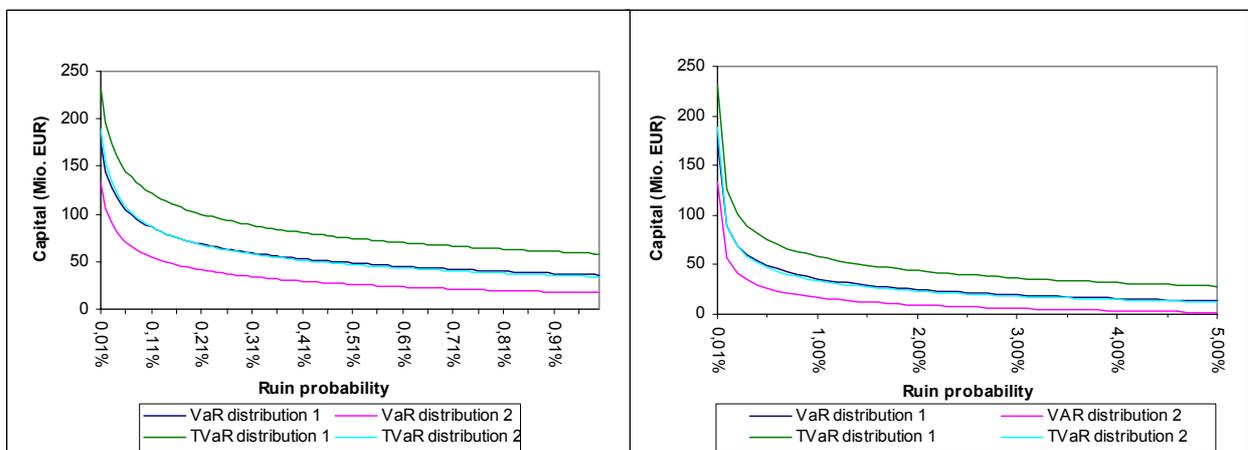


Figure 3-3: Capital requirements with varying ruin probabilities

With decreasing security level the risk adequate capital decreases as well. But for very small ruin probabilities the risk adequate capital decreases super proportional. The next table shows the effect of increasing the ruin probability by steps of 0.1%.

Increasing ϵ from ϵ_1 to ϵ_2	Distribution 1				Distribution 2			
	VaR		TVaR		VaR		TVaR	
0,1% - 0,2%	-18,83 Mio. €	-21,27%	-23,71 Mio. €	-18,94%	-15,11 Mio. €	-26,39%	-20,44 Mio. €	-22,85%
0,2% - 0,3%	-9,74 Mio. €	-13,98%	-12,36 Mio. €	-12,18%	-7,55 Mio. €	-17,92%	-10,33 Mio. €	-14,97%
0,3% - 0,4%	-6,38 Mio. €	-10,65%	-8,14 Mio. €	-9,13%	-4,84 Mio. €	-13,99%	-6,67 Mio. €	-11,36%
0,4% - 0,5%	-4,67 Mio. €	-8,71%	-5,97 Mio. €	-7,38%	-3,47 Mio. €	-11,68%	-4,82 Mio. €	-9,26%
0,5% - 0,6%	-3,63 Mio. €	-7,43%	-4,67 Mio. €	-6,22%	-2,67 Mio. €	-10,16%	-3,72 Mio. €	-7,88%
0,6% - 0,7%	-2,95 Mio. €	-6,52%	-3,80 Mio. €	-5,40%	-2,14 Mio. €	-9,07%	-3,00 Mio. €	-6,90%
0,7% - 0,8%	-2,47 Mio. €	-5,84%	-3,19 Mio. €	-4,79%	-1,77 Mio. €	-8,26%	-2,49 Mio. €	-6,16%
0,8% - 0,9%	-2,11 Mio. €	-5,30%	-2,73 Mio. €	-4,31%	-1,50 Mio. €	-7,63%	-2,12 Mio. €	-5,58%
0,9% - 1,0%	-1,84 Mio. €	-4,87%	-2,38 Mio. €	-3,93%	-1,30 Mio. €	-7,13%	-1,84 Mio. €	-5,12%
1,0% - 1,1%	-1,62 Mio. €	-4,52%	-2,11 Mio. €	-3,62%	-1,14 Mio. €	-6,73%	-1,61 Mio. €	-4,74%
1,1% - 1,2%	-1,45 Mio. €	-4,22%	-1,88 Mio. €	-3,36%	-1,01 Mio. €	-6,39%	-1,43 Mio. €	-4,42%
1,2% - 1,3%	-1,30 Mio. €	-3,97%	-1,70 Mio. €	-3,13%	-0,90 Mio. €	-6,11%	-1,29 Mio. €	-4,15%
1,3% - 1,4%	-1,18 Mio. €	-3,76%	-1,55 Mio. €	-2,94%	-0,81 Mio. €	-5,88%	-1,16 Mio. €	-3,91%
1,4% - 1,5%	-1,08 Mio. €	-3,57%	-1,41 Mio. €	-2,77%	-0,74 Mio. €	-5,68%	-1,06 Mio. €	-3,71%
1,5% - 1,6%	-1,00 Mio. €	-3,40%	-1,30 Mio. €	-2,63%	-0,68 Mio. €	-5,50%	-0,97 Mio. €	-3,53%
1,6% - 1,7%	-0,92 Mio. €	-3,26%	-1,21 Mio. €	-2,50%	-0,62 Mio. €	-5,36%	-0,90 Mio. €	-3,38%
1,7% - 1,8%	-0,85 Mio. €	-3,12%	-1,12 Mio. €	-2,38%	-0,57 Mio. €	-5,23%	-0,83 Mio. €	-3,24%
1,8% - 1,9%	-0,80 Mio. €	-3,01%	-1,05 Mio. €	-2,28%	-0,53 Mio. €	-5,12%	-0,77 Mio. €	-3,11%

Figure 3-4: Effects of an increasing ruin probability on the risk adequate capital

The additional capital for relative small increases of the security level is rather high for the focused security levels. Hence, given a constant cost of capital rate the cost of capital increases super proportional with the security level as well. The following figure shows this effect for different cost of capital rates (c) based on the Value-at-Risk of distribution one:

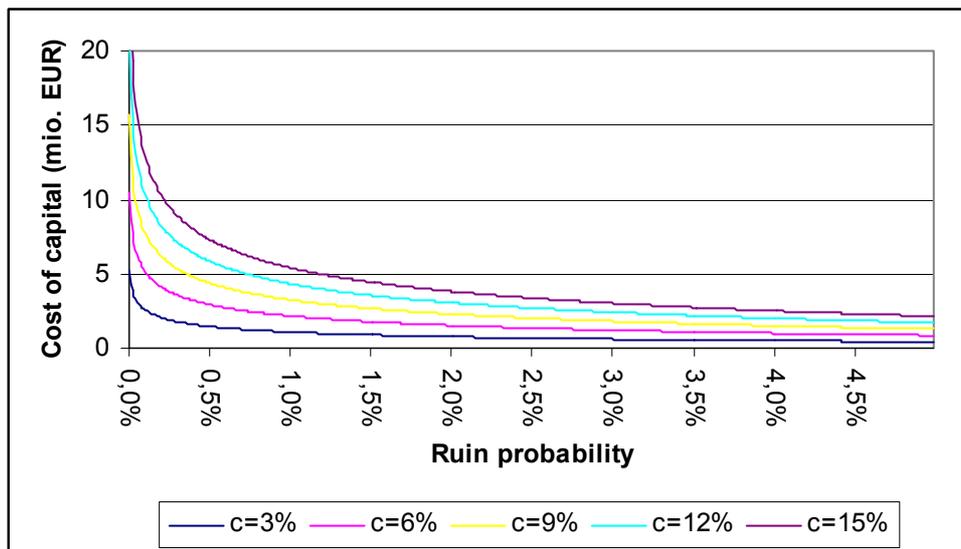


Figure 3-5: Cost of capital

In chapter 5 we'll confront this cost of capital-effect with the premium-effect which is described in the next section.

4. The premium-effect

To explain the behaviour of policyholders by decision theory there compete normative and descriptive models. The probably most prominent normative decision theory is the Expected Utility Theory. Assessing normative decision models is quite difficult because they are based on an ex ante defined rationality of the decision maker, which is reflected in their axiomatization.¹² *Wakker/Thaler/Tversky* (1997) demonstrate by using the Expected Utility Theory that the premiums for probabilistic insurance approximately coincide with the actuarial adjusted premiums for standard insurance.¹³ That means to weight the standard premium with the security level. That doesn't depend on the degree of risk aversion.¹⁴

$$(6) \quad \pi^{\text{PI}} = (1 - \varepsilon)\pi^{\text{SI}}$$

The axiomatic of Expected Utility Theory is object of a lot of empirical studies. These show that individuals doesn't behave as the Expected Utility Theory predicts.¹⁵ So, we have to refuse its explanation of policyholders' decision making. In fact, these studies show that the willingness to pay for probabilistic insurance decreases super proportional with the security level.

The discrepancy between rational and empirical behaviour is causal for the development of descriptive decision models. These models are based on empirical insights.¹⁶ The most prominent descriptive decision theory is the Prospect Theory by *Kahneman/Tversky* (1979). The Prospect Theory describes the individual decision making in two phases.

First, the decision problem has to be framed and edited. That means in particular to determine the reference point and to simplify the problem with defined editing

¹² See Bamberg/Trost 1996, pp. 642.

¹³ See in detail Wakker/Thaler/Tversky 1997, pp. 22.

¹⁴ See Maurer 2000, p. 124; Wakker/Thaler/Tversky 1997, p. 12.

¹⁵ See in detail Kahneman/Tversky 1979, pp. 265.

¹⁶ See Bamberg/Trost 1996, S. 642.

operations. But these operations don't lead to an unambiguous result. The bare variation of the operation sequence may lead to different results.¹⁷ The determination of the reference point is of vital importance. However, a generalization isn't trivial either. Indeed, *Wakker/Thaler/Tversky* (1997) as well as *Maurer* (2000) determine the reference point with the policyholders' current asset position.¹⁸ However, other reference points are feasible.¹⁹

Secondly, the possible states have to be valuated. Therefore, *Kahneman/Tversky* (1979) develop a value function depending on a reference point. Hence, gains and losses rather than final asset positions are valuated. An essential implication of Prospect Theory is the concavity of the value function for gains and its convexity for losses.²⁰ Nevertheless, the convexity of the value function for losses is not indisputable in the insurance context. Indeed, a study of *Hershey/Schoemaker* (1980) confirms the decreasing demand of insurance with growing claims and therefore a convexity of the value function of losses.²¹ However, *Ganderton et al.* (2000) show that this depends less on claim heights than on low occurrence probabilities.²² *Laughunn/Payne/Crum* (1980) suppose a risk aversion for very high losses.²³ But the following analysis focuses on the ruin probability. Therefore, a closer consideration of the value function is not necessary.

Regarding the ruin probability as a risk measure for the determination of risk adequate capital, the perception of probability by policyholders is obviously important. *Kahneman/Tversky* (1979) describe this perception with the probability weighting function.²⁴ The certainty effect shows the subadditivity of probability weights.²⁵ So,

¹⁷ See in detail Theil 2002, pp. 131.

¹⁸ See Maurer 2000, pp. 135; Wakker/Thaler/Tversky 1997, pp. 15.

¹⁹ See Kahneman/Tversky 1979, p. 274.

²⁰ It's also steeper for losses than for gains. See Kahneman/Tversky 1979, p. 279.

²¹ See Hershey/Schoemaker 1980, pp. 399.

²² See Ganderton et al. 2000, pp. 278.

²³ See Laughunn/Payne/Crum 1980, pp. 1245.

²⁴ The following description of the probability weighting function refers to the Cumulative Prospect Theory (CPT) by Tversky/Kahneman 1992.

²⁵ This effect describes, that individuals overweight gains that they perceive as certain.

certain events have a special relevance.²⁶ Thus, this effect supports the attractiveness of insurance cover in absence of default risk.

The probability weighting function $\varphi_{\text{CPT}}(p)$ describes the overweighting of small probabilities and the underweighting of medium and high probabilities. The threshold between over- and underweighting is identified by *Wu/Gonzalez* (1996) at $p \approx 0,4$.²⁷ With respect to insurance supervision, we focus on small ruin probabilities. Hence, the relevant probabilities of ruin are over weighted.²⁸ Furthermore, there are different curves for gains and losses, especially for high probabilities.²⁹

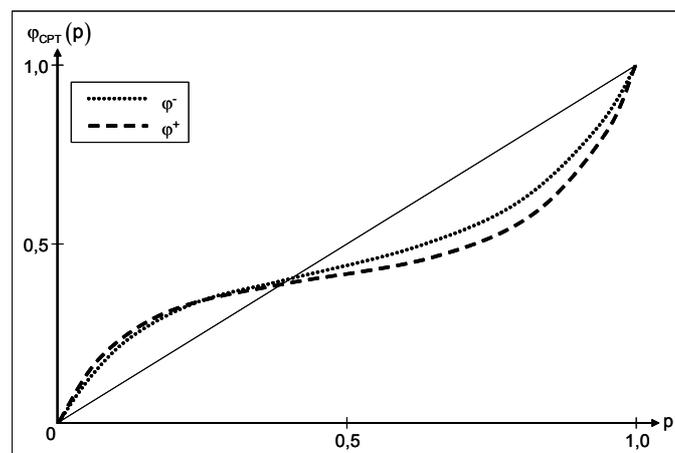


Figure 4-1: Probability weighting function³⁰

From empirical studies *Tversky/Kahneman* (1992) deduce the following equations of the probability weighting function for gains respectively losses.³¹

$$(7a) \quad \varphi_{\text{CPT}}^+(p) = \frac{p^\psi}{(p^\psi + (1-p)^\psi)^{1/\psi}}$$

²⁶ See *Kahneman/Tversky* 1979, pp. 280; *Theil* 2002, p. 165.

²⁷ See *Wu/Gonzalez* 1996, pp. 1682 and *Prelec* 1998, pp. 505.

²⁸ See *Theil* 2002, p. 242.

²⁹ See *Tversky/Kahneman* 1992, pp. 312.

³⁰ See *Tversky/Kahneman* 1992, p. 313.

³¹ See *Tversky/Kahneman* 1992, p. 309.

$$(7b) \quad \varphi_{\text{CPT}}^-(p) = \frac{p^\phi}{(p^\phi + (1-p)^\phi)^{1/\phi}}$$

Tversky/Kahneman (1992) estimate the relevant parameters with $\psi = 0,61$ and $\phi = 0,69$.³² To describe the perception of the ruin probability by the policyholders we focus on the probability weighting function for losses. The following figure illustrates the weighting of low ruin probabilities:

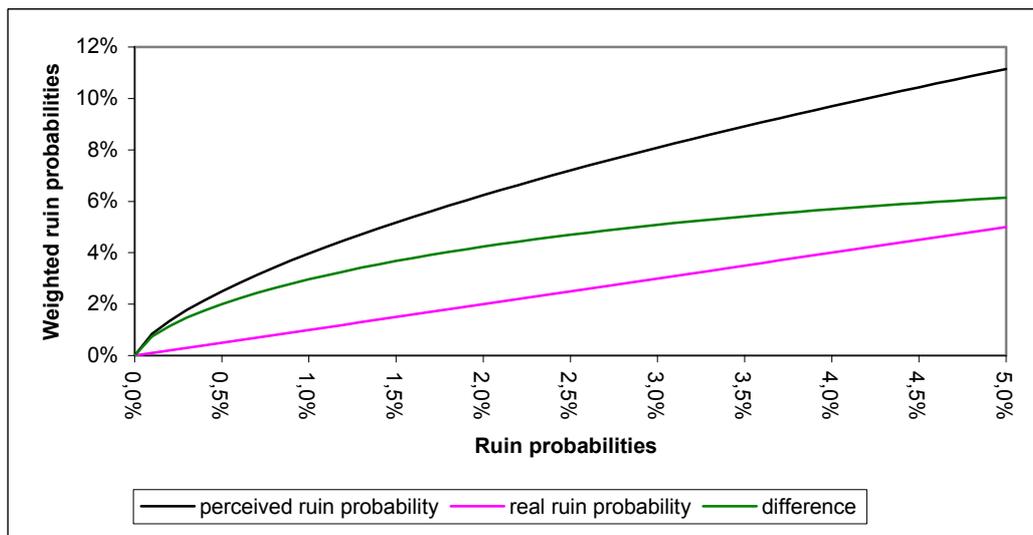


Figure 4-2: Ruin probability weighting

The figure shows the overweighting of the real ruin probability. Thus, given a target probability of ruin, it's important to make a difference between real and perceived probabilities. The following table compares the capital requirement based on real ruin probabilities with the capital requirement based on perceived ruin probabilities.

³² See Tversky/Kahneman 1992, pp. 311. The study of *Wu/Gonzalez* (1996) approximately confirms this estimation. See *Wu/Gonzalez* 1996, pp. 1682.

ruin probability	capital requirement based on real r.p.	capital requirement based on perceived r.p.	additional capital requirement	
			in mio. EUR	in %
0,1%	88,5 Mio. €	215,0 Mio. €	126,5 Mio. €	142,9%
0,2%	69,7 Mio. €	165,3 Mio. €	95,6 Mio. €	137,2%
0,3%	60,0 Mio. €	140,3 Mio. €	80,3 Mio. €	134,0%
0,4%	53,6 Mio. €	124,3 Mio. €	70,7 Mio. €	132,0%
0,5%	48,9 Mio. €	112,7 Mio. €	63,8 Mio. €	130,5%
0,6%	45,3 Mio. €	103,8 Mio. €	58,5 Mio. €	129,2%
0,7%	42,3 Mio. €	96,6 Mio. €	54,3 Mio. €	128,4%
0,8%	39,8 Mio. €	90,7 Mio. €	50,9 Mio. €	127,7%
0,9%	37,7 Mio. €	85,7 Mio. €	47,9 Mio. €	127,1%
1,0%	35,9 Mio. €	81,3 Mio. €	45,4 Mio. €	126,5%
1,1%	34,3 Mio. €	77,5 Mio. €	43,2 Mio. €	126,1%
1,2%	32,8 Mio. €	74,1 Mio. €	41,3 Mio. €	125,7%
1,3%	31,5 Mio. €	71,1 Mio. €	39,5 Mio. €	125,5%
1,4%	30,3 Mio. €	68,3 Mio. €	38,0 Mio. €	125,2%
1,5%	29,3 Mio. €	65,8 Mio. €	36,6 Mio. €	125,0%
1,6%	28,3 Mio. €	63,5 Mio. €	35,3 Mio. €	124,8%
1,7%	27,3 Mio. €	61,4 Mio. €	34,1 Mio. €	124,7%
1,8%	26,5 Mio. €	59,5 Mio. €	33,0 Mio. €	124,5%
1,9%	25,7 Mio. €	57,7 Mio. €	32,0 Mio. €	124,4%

Figure 4-3: Effect of probability weighting on capital requirements

E.g. the insurance company needs 35.9 Mio. € to achieve a real ruin probability of 1%, but 81.3 Mio. € to achieve the same perception by the policyholders.³³ Nevertheless, that kind of view doesn't fit to our model, because the probability perception doesn't matter itself, but its effect on the policyholders' willingness to pay for probabilistic insurance. The figure assumes constant premiums of 10 Mio. € for standard insurance. Because of probabilistic insurance, these premiums have to be reduced corresponding with the ruin probability. *Wakker/Thaler/Tversky* (1997) quantify the premiums for probabilistic insurance by:³⁴

$$(8) \quad \begin{aligned} \pi^{\text{Pl}} &= (1 - \varphi(\varepsilon)) \cdot \pi^{\text{Sl}} \\ &= (1 - \varepsilon) \cdot \pi^{\text{Sl}} - (\varphi(\varepsilon) - \varepsilon) \cdot \pi^{\text{Sl}} \end{aligned}$$

This equation contains two components. The first component describes the actuarial reduced premiums. As mentioned this equals the premium reduction predicted by the Expected Utility Theory. The second component reflects the difference between perceived and real ruin probability. Therefore, the premium reduction following Prospect Theory exceeds the reduction *Wakker/Thaler/Tversky* (1997) determined

³³ A perceived ruin probability of 1% equals a real ruin probability of 0.129%. See equation (7b).

³⁴ See *Wakker/Thaler/Tversky* 1997, p. 16.

for Expected Utility Theory. Based on a premium of 10 Mio. EUR for standard insurance, the following table shows these components.

Ruin probability	premium - Expected Utility Theory	additional risk adjustment	premium - Prospect Theory
0,1%	9,990 Mio. €	0,074 Mio. €	9,916 Mio. €
0,2%	9,980 Mio. €	0,115 Mio. €	9,865 Mio. €
0,3%	9,970 Mio. €	0,147 Mio. €	9,823 Mio. €
0,4%	9,960 Mio. €	0,175 Mio. €	9,785 Mio. €
0,5%	9,950 Mio. €	0,200 Mio. €	9,750 Mio. €
0,6%	9,940 Mio. €	0,223 Mio. €	9,717 Mio. €
0,7%	9,930 Mio. €	0,243 Mio. €	9,687 Mio. €
0,8%	9,920 Mio. €	0,262 Mio. €	9,658 Mio. €
0,9%	9,910 Mio. €	0,280 Mio. €	9,630 Mio. €
1,0%	9,900 Mio. €	0,297 Mio. €	9,603 Mio. €
1,1%	9,890 Mio. €	0,312 Mio. €	9,578 Mio. €
1,2%	9,880 Mio. €	0,327 Mio. €	9,553 Mio. €
1,3%	9,870 Mio. €	0,341 Mio. €	9,529 Mio. €
1,4%	9,860 Mio. €	0,355 Mio. €	9,505 Mio. €
1,5%	9,850 Mio. €	0,368 Mio. €	9,482 Mio. €
1,6%	9,840 Mio. €	0,380 Mio. €	9,460 Mio. €
1,7%	9,830 Mio. €	0,391 Mio. €	9,439 Mio. €
1,8%	9,820 Mio. €	0,403 Mio. €	9,417 Mio. €
1,9%	9,810 Mio. €	0,413 Mio. €	9,397 Mio. €
2,0%	9,800 Mio. €	0,424 Mio. €	9,376 Mio. €

Figure 4-4: Willingness to pay for probabilistic insurance

The table shows an increasing premium reduction for an increasing ruin probability. Thus, the policyholders' willingness to pay for probabilistic insurance increases super proportional with decreasing ruin probabilities.

5. Comparison of cost of capital-effect and premium-effect

To identify the value maximizing security level the insurance management has to compare the cost of capital-effect and the premium-effect. The maximum of the premiums reduced by the cost of capital determines the highest value proposition.

Continuing our example, we assume a constant cost of capital rate given and independent from the ruin probability. The following figure (5-1) shows the described effects for different cost of capital rates (c).

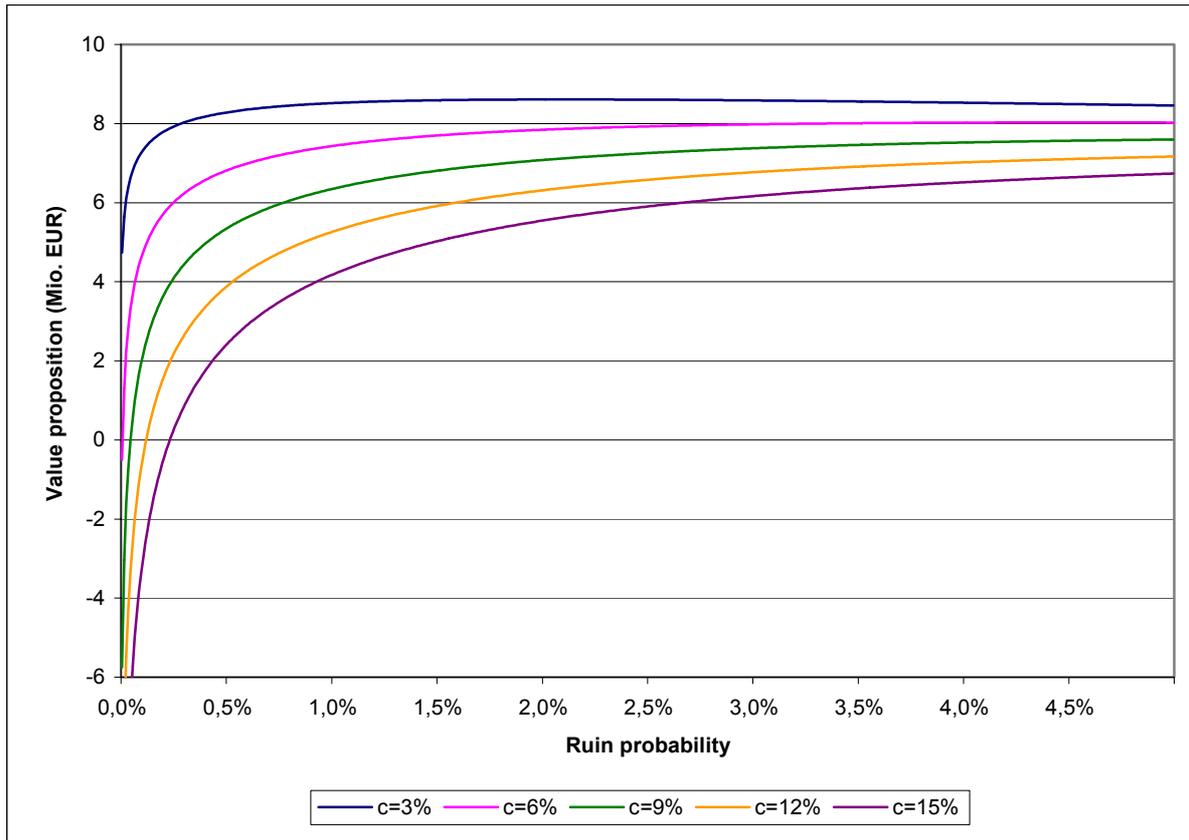


Figure 5-1: Premiums reduced by cost of capital

The figure shows the value proposition for different ruin probabilities. The capital requirements base on real ruin probabilities. But the premiums influence the capital requirements as well. A reduction of the security level decreases the capital requirement but also the policyholders' willingness to pay. Because both, the capital and the premiums determine the security level the decreasing premiums reduce the cost of capital reduction.

According to insurance supervision, our analysis contains only ruin probabilities up to 5%. The figure decomposes positive and negative value propositions. For instance, in our example the thresholds are approximately $\varepsilon \approx 0,24\%$ for $c=15\%$ and $\varepsilon \approx 0,12\%$ for $c=12\%$. Ruin probabilities below these thresholds cause a negative value proposition, whereas ruin probabilities above these thresholds cause a positive value proposition. Within the model, it is possible to identify the value maximizing security levels. For instance they are at $\varepsilon \approx 2,10\%$ for $c=3\%$ and $\varepsilon \approx 4,58\%$ for $c=6\%$. The value maximizing security levels for the other cost of capital rates exceed

the focused area of ruin probabilities up to 5%. But the description of the policyholders' willingness to pay is not that accurate to identify these points in reality.

6. Conclusion

The present paper has lit up the capital budgeting decision of insurance companies with respect to value based management. In this context, we have focused the choice of the security level. To simplify the problem we made considerable assumptions. For instance, the security level does not only depend on the capital budget, but is also determined by underwriting policy, reinsurance and other influences. For reasons of complexity reduction, the modelling of the capital requirements is rather pragmatic as well.

Also the description of the policyholders' willingness to pay implies problems. This is in particular the generalization of empirically observed patterns of behaviour. Indeed, the Prospect Theory explains the empirical findings about the policyholders' willingness to pay much better than the Expected Utility Theory. But, the Prospect Theory is too ambiguous to exactly point out the premium-effect in reality.

Nevertheless, the analysis generates important knowledge. Changes of the security level influence different cash flows. Regarding supervisory minimum requirements the insurance management has to identify and quantify the relevant effects of changing security levels to fix the security level in a value enhancing way. The analysis above has shown this in an example. As seen, the value enhancing security level is as lower as higher the cost of capital rate is. As this means that the capital becomes more expensive accordingly, this seems also reasonable.

The underlying assumptions, as well as the need for more accurate instruments for the description of both effects, in particular the premium-effect, motivate further research.

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